SCHOOL OF THE WEST

Electronics

Chapter 7: Kirchhoff's laws

Maths Review

Systems of equations

- When you have more than one equation related to each other, you have a system of equations.

- There are many ways to solve it: **substitution**, elimination, graphing, matrix solution (Gauss method)...

- We need at least as many equations as we have variables.

For example:

$2x + 3y - 8z + 85 = 0$

$8x + 54y - z = 0$

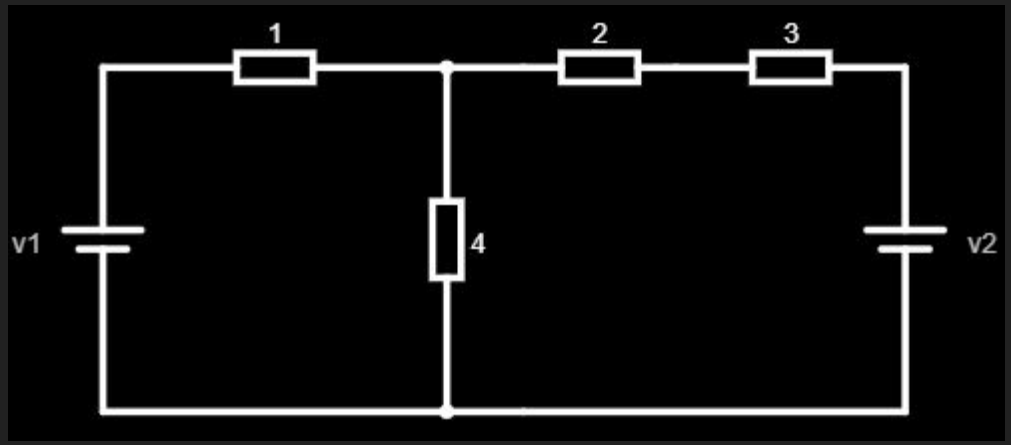
$y - 3z/2 + 10 = 0$

Concepts

Kirchhoff's laws

- Current Law: The directed sum of currents in a line junction is zero.
- Voltage Law: The directed sum of voltages around a closed loop is zero.
- Postulated by Gustav Kirchhoff
- Allows us to calculate all currents and voltages in any circuit

Kirchhoff - Problem introduction

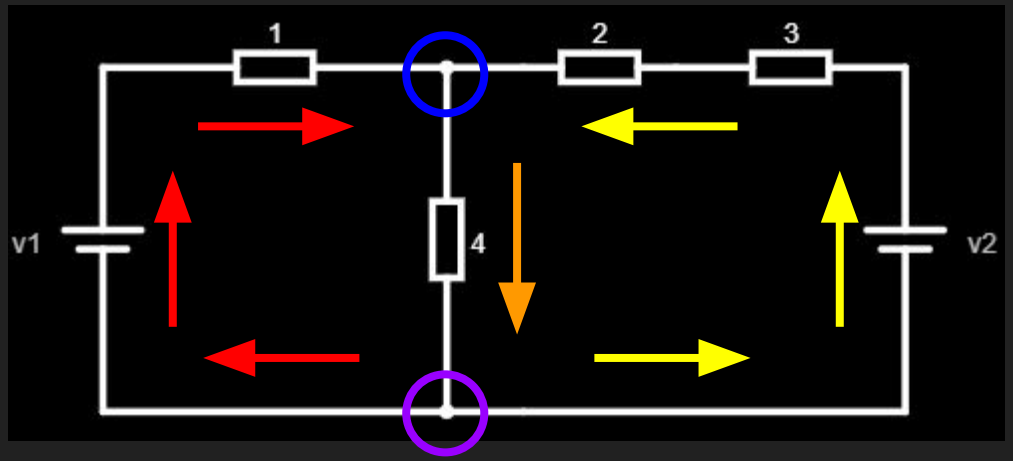


V1: 16V R1: 4Ω R3: 2Ω

V2: 8V R2: 2Ω R4: 8Ω

- Given this circuit, we want to know the current at R2.
- We could add R2 and R3 as they are in series, but what do we do with R4? And what do we do with the multiple batteries?
- To solve this, we need to use Kirchhoff's laws.

Kirchhoff - Identify junctions and currents

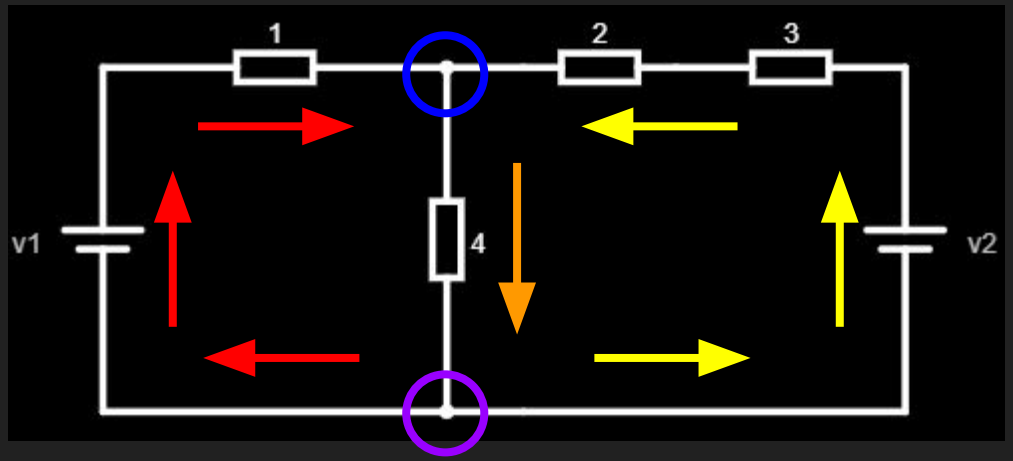


V1: 16V R1: 4Ω R3: 2Ω

V2: 8V R2: 2Ω R4: 8Ω

- We identify junctions of the circuit, we will call them **J1** and **J2**
- We identify the currents and we decide their direction. We will call them **I1**, **I2** and **I3**. Notice they go from junction to junction.

Kirchhoff - Get Junction equations



V1: 16V R1: 4Ω R3: 2Ω

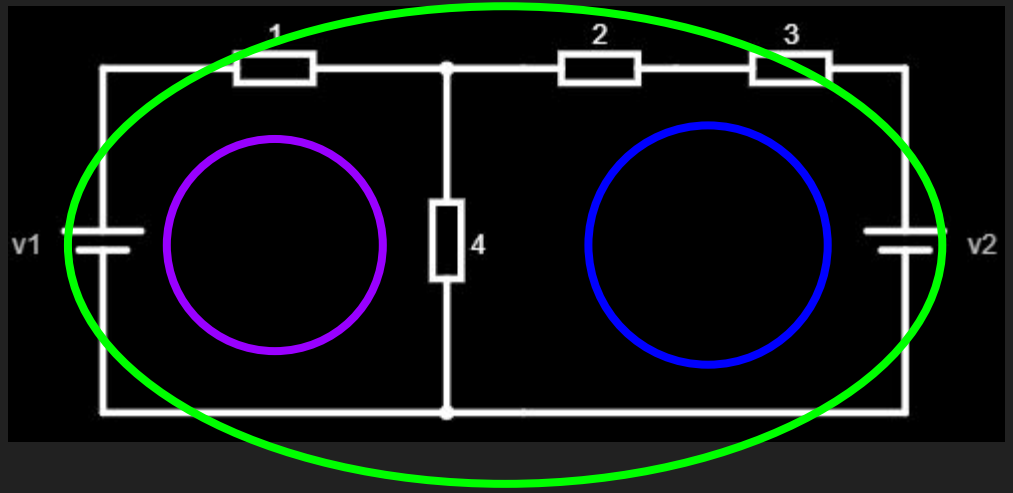
V2: 8V R2: 2Ω R4: 8Ω

- Using the current law, we know that currents entering a junction minus currents leaving it equal 0.
- We get these equations:

J1: **I1** + **I3** - **I2** = 0

J2: **I2** - **I1** - **I3** = 0

Kirchhoff - Identify loops



V1: 16V R1: 4Ω R3: 2Ω

V2: 8V R2: 2Ω R4: 8Ω

- We identify closed loops of the circuit, we will call them **L1**, **L2** and **L3**.
- We will decide that all loops flow clockwise.

Kirchhoff - Get loop equations - 1

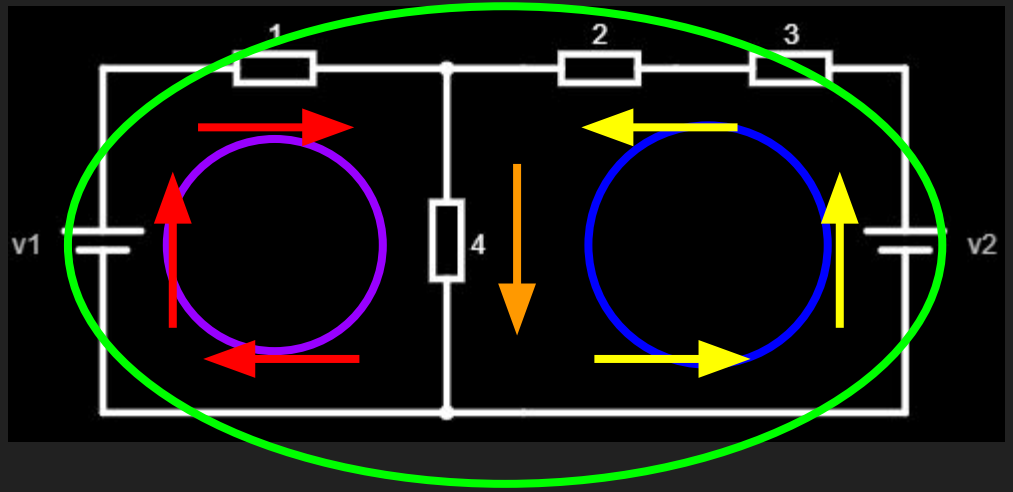
- Using the loop law, we know that the sum of directed voltages in a loop is equal to 0.

- Batteries that go in the loop direction have a positive voltage
- Batteries that go against the loop direction have a negative voltage

- By Ohm’s law we know a resistor has a voltage equal to I * R.

- Resistors whose current goes in the loop direction have a negative voltage
- Resistors whose current goes against the loop direction have a positive voltage

Kirchhoff - Get loop equations - 2



V1: 16V R1: 4Ω R3: 2Ω

V2: 8V R2: 2Ω R4: 8Ω

- We get these equations:

L1: +16 -4I1 -8I2 = 0

L2: +8I2 +2I3 +2I3 -8 = 0

L3: +16 -4I1 +2I3 +2I3 -8 = 0

Kirchhoff - Solve the system of equations - 1

- We have a system of 5 equations and 3 variables:

  J1: I1 + I3 - I2 = 0

  J2: I2 - I1 - I3 = 0

  L1: +16 -4I1 -8I2 = 0

  L2: +8I2 +2I3 +2I3 -8 = 0

  L3: +16 -4I1 +2I3 +2I3 -8 = 0

- We are going to solve it by substitution using 3 of these equations. There are many possible ways to solve it, try to pick the one you think will be easier.

Kirchhoff - Solve the system of equations - 2

- We will start solving L1 for **I1**

+16 -4I1 -8I2 = 0

-4I1 = +8I2 -16

4I1 = -8I2 +16

I1 = (+16 - 8I2)/4

I1 = +4 -2I2

Kirchhoff - Solve the system of equations - 3

- Then we will solve J1 for **I3**, replacing **I1** by the expression we got in previous step (**I1** = +4 -2**I2**)

+I1 +I3 -I2 = 0

+4 -2I2 +I3 -I2 = 0

+4 -3I2 +I3 = 0

I3 = +3I2 -4

Kirchhoff - Solve the system of equations - 4

- Finally we will solve L2 for **I2**, replacing **I3** by the expression we got in previous step (**I3** = +3**I2** -4)

+8I2 +2I3 +2I3 -8 = 0

+8I2 +4I3 -8 = 0

+8I2 +4(+3I2 -4) -8 = 0

+8I2 +12I2 -16 -8 = 0

+20I2 -24 = 0

I2 = 24/20

I2 = 1.2

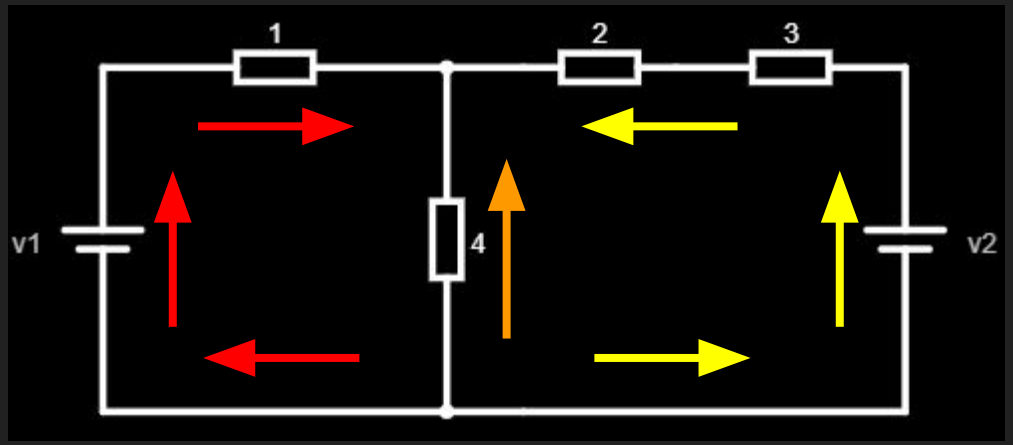
Kirchhoff - Solve the system of equations - 5

- Now that we know the value for **I2** (1.2) we just need to use the expressions for **I1** and **I3**

I3 = +3I2 -4	**I1 = +4 -2I2**
I3 = +3*1.2 -4	**I1 = +4 -2*1.2**
I3 = -0.4	**I1 = 1.6**

- Notice **I3** is negative, this means we got the wrong direction.

Kirchhoff - Solved!



V1: 16V V2: 8V

R1: 4Ω R3: 2Ω

R2: 2Ω R4: 8Ω

I1: 1.6A

I2: 1.2A

I3: 0.4A

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End of first module

Next module: DC electronics advanced